Title: **Sharp Interface And Voltage Conservation In The Phase Field Method: Application To Cardiac Electrophysiology**

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Cardiac Electrophysiology is one of the applications that benefited from the development of phase-field methods. In these methods the original equations are modified by introducing a smooth phase field function ϕ that describe the boundary of the physical domain and then solved in a larger domain. The region of interest is characterized by setting $\phi > 0$ (typically $\phi = 1$ on a large portion of the region) and $\phi = 0$ outside. The region $\{0 < \phi < 1\}$ is called transition layer or diffuse interface. The width of this layer can be made as small as needed for each particular situation. A list of applications of the phase-field method can be found in [1] and a detailed description of the method in the context of cardiac simulation is given in [2].

In the area of electrophysiology the governing equation is the continuous cable equations for the voltage: $\frac{\partial V}{\partial t} = \nabla \cdot D\nabla V - I_{ion}$, where D is a 3 by 3 diffusion tensor of conductivity. This tensor carries the information of fiber orientation which, in general is not perpendicular to the boundary of the region of interest. If Neumann boundary conditions are cosnidered, because there is no current flux through the boundary, the voltage must be conserved in the absence of the i_{ion} source term.

In this paper, the authors take advantage of that fact and by integrating the modified (by the phase-field function) cable equations and going to the limit when the diffuse layer has zero thickness after discretization, they recovered a conservative approximation that can be used for arbitrary geometries.

Thus, a novel finite differences scheme to evaluating the non-ionic part of the modified cable equation $\phi\frac{\partial V}{\partial t} = \nabla \cdot (\phi D\nabla V) - \phi I_{ion}$ is presented in this work. Numerical examples are given for benchmark problems and for the case of a rabbit ventricle using real physiological data. Comparison with similar approaches, show very good results.

References:

1. Fenton, F.h., Cherry, E. M., Karma, A. and Rappel, W. *Modelling wave propagation in realistic heart geometries using the field-phase method*, Chaos **15**, 013502 (2005)

2. Anderson, D. *Diffuse-Interface and Phase-Field Modeling* available online at:
`http://math.gmu.edu/ dmanders/WEBDAN/diffuse.html#phase2`

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