Title: **A singular perturbation of the heat equation with memory**

Author: **Branco, J.R., and Ferreira, J.A.**

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Review by: Mario Forcinito

The paper addresses the solution of Initial Boundary Value Problem:

$$\epsilon\frac{\partial^2 u}{\partial t^2}(x,t)+\alpha\frac{\partial u}{\partial t}(x,t)=\gamma\frac{\partial^2 u}{\partial x^2}(x,t)+\int_0^t k(t-s)\frac{\partial^2 u}{\partial x^2}(x,s)ds+f(x,t,u(x,t)) \quad (1)$$

$$\begin{cases} u(x,0)=u_0(x), & x\in(a,b) \\ \frac{\partial u}{\partial t}(x,0)=u_1, & x\in(a,b) \end{cases} \quad (2)$$

$$u(a,t)=u_a(t),\ u(b,t)=u_b(t),\ t>0 \quad (3)$$

This equation arises as the mathematical model of several systems in engineering and physics, i.e., linear viscoelasticity.

The analysis is done by analyzing the solutions to:

$$\alpha\frac{\partial w}{\partial t}(x,t)=\gamma\frac{\partial^2 w}{\partial x^2}(x,t)+\int_0^t k(t-s)\frac{\partial^2 w}{\partial x^2}(x,s)ds+f(x,t) \quad (4)$$

that can be considered as a singular perturbation of (1)-(3). In particular the authors consider the case in which:

$$k(s)=\frac{\sigma}{\tau}e^{-s/\tau} \quad (5)$$

of particular interest for the analysis of the heat eqaution with memory.

Discrete versions of (1)-(3) and (4) are developed by and the behaviour and stability of the solutions analyzed over a range of the parameters ϵ,σ,τ and γ.

Related:

- Christoforou, C., Systems of Hyperbolic Conservation Laws with Memory.
  `http://www.math.ntnu.no/conservation/2006/047.pdf/`