Title: **Second-Order Slope Limiters for the Simultaneous Linear Advection of (not so) Independent Variables**

Author: **Tran, Q. H.**

Published in: Communications in Mathematical Sciences, **6**, No. 3, pp 569–593, 2008.

Review by: Mario Forcinito

Mathematical modeling of industrial applications (multi-phase and multi-species flow with chemical interactions, typically modeled using the Arbitrary Lagrange-Euler formalism, for example) involve the solution of systems of linear advection equations for which the independent variables are related, at a discrete level, by a set of physically meaningful control variables. The author proposes and analyzes a strategy that ensures a discrete min-max principle for the main variables and the control variables and preserves a high order of accuracy. In the context of this paper, control variables can, in general, be first order rational functions of the main variables.

The strategy is based on heuristic rules explained by the author in geometrical terms for the sum problem (that in which the control variable is the sum of all the main variables). An heuristic rule is formulated, based on splitting the variables and using localized slope limiters. The numerical results seem to validate the heuristic rule and preserve the accuracy at a very good level, which the author calls the a 'miracle'.

The presentation is clear and fairly detailed. Several examples are described for uniform and variable velocity fields.

This is a relatively unexplored path within the field. Without doubt, the strategy presented in this paper will open the way to further research and refinements useful for practical computational purposes.