Title: **A time-adaptive finite volume method for the Cahn-Hilliard and Kuramoto-Sivashinsky equations**

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There are several scientific and industrial applications for the Cahn-Hilliard equations, such as phase separation after quenching of binary alloys, interfacial fluid flow and polymer science. Studying the phase turbulence in the Belousov-Zhabotinsky reaction, Kuramoto and, independently, working in the context of small thermal diffusive instabilities for laminar flame fronts Sivashinsky, introduced the Kuramoto-Sivashinsky equation.

Cahn-Hilliard and Kuramoto Sivashinsky type of equations have have been used to model directional solidification and weak fluid turbulence.

Due to their high order and complex behaviour, solution of Cahn-Hilliard and Kuramoto Sivashinsky type equations represent a formidable challenge to numerical methods. In this work, the authors present a complete finite volume based method, that combines a stable, high-order accurate spatial discretization together with an adaptive time-stepping based on Runge-Kutta pairs for stiff ODE's. The time step selection strategy is based on the control of error.

The spatial discretization is general enough to cover a finite volume (unstructured) with a stable and high-order approximation. The authors describe the method in detail and give indication on how to treat the boundaries for a general case.

Different implicit-explicit Runge-Kutta method are analyzed and adapted for the time integration. The authors examine several variations of methods. These are supplemented with time-step controller criteria based on the local error with the objective of keeping the error close to a certain tolerance.

The paper is carefully written, detailed and the objectives clearly stated. Numerical results are presented for 1-D and 2-D representative problems of both equations. Results shown demonstrate that accurate quantitative solutions can be obtained using the proposed methodology.

The method presented in this work will certainly open the door to the effective simulation of of complex phenomena related to phase-ordering, coarsening and systems that exhibit chaotic behaviour or mild turbulence.

See also:

1. Boghosian, B. M. et al. *Hydrodynamics of the Kuramoto-Sivashinsky Equation in Two Dimensions*, Phys. Rev. Let., (**83**), No 25, available on-line at:
   `http://matisse.ucsd.edu/ hwa/pub/ks2d.pdf`

2. Ascher, U. M. et al. *Implicit-Explicit Runge-Kutta Methods for Time-Dependent Partial Diferential Equations* available on-line at:
   `ftp://ftp.math.ucla.edu/pub/camreport/cam97-10.ps.gz`

3. Papers on Cahn-Hilliard equation by Prof. Elliot available on-line at:
   `http://www.warwick.ac.uk/staff/C.M.Elliott/cahnhilliard.html`