Title: **A Cartesian Embedded Boundary Method for the Compressible Navier-Stokes Equations**

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The Cartesian Embedded Boundary Method (CEBM) consist of a numerical scheme for the solution of a set of differential equations on a regular cartesian grid (typically a second order finite difference method) over which the geometry is superimposed in the form of no-computation cells with ad-hoc interpolation of the values to represent the exact boundary. In practical applications, the cells surrounding the embedded geometry, or internal discontinuity surfaces, can be further subdivided to better fit the geometry (adaptive mesh refinement).

CEBMs exhibit a consistent high order approximation together with an extreme simplicity in terms of grid generation and relatively small memory requirements. However, the ability to deal with complex boundaries is based on finding the optimal scheme for the interpolation of values at the cartesian grid points surrounding the boundary, where a Dirichlet or Neumann condition is specified. The optimal interpolation is understood to be one such that the order and the accuracy of the approximation in the cartesian grid is maintained at the embedded boundaries.

This is easier said than done as there is no known universal interpolation method for the general case. Moreover, different terms of the same equation may have different optimal interpolation schemes.

In this paper, the authors assessed the validity and accuracy of two interpolation philosophies, the one proposed by Kreiss & Petersson[1] (KP) and the one proposed by Sjögreen & Petersson[2] (SP). These are applied to the compressible Navier-Stokes equations with Dirichlet and Neumann boundary conditions.

Several examples are presented and results compared to solutions obtained using body fitted meshes. The comparisons show that the overall second order accuracy is maintained (although there is no rigorous proof).

The results also show that they are comparable to those obtained by the body fitted mesh if the aspect ratio of the refined grid are close to 1.

Computational advantages of embedded boundary methods with respect to body fitted meshes are expected for 3-D problems with complex geometries. Also the ability to use higher order accurate finite difference methods, is another advantage with respect to the first order accuracy typically obtained by using body fitted meshes or unstructured grids.

References:

1. Kreiss, H.O. and Petersson, A. *A Second Order Accurate Embedded Boundary Method For The Wave Equation With Dirichlet Data*, SIAM J. SCI. COMPUT. **27**, No. 4, pp. 1141-1167, (2006), available on-line at:
   `https://computation.llnl.gov/casc/serpentine/pubs/KP-Dirichlet-SISC.pdf`

2. Sjögreen, B. and Petersson, A. *A Cartesian Embedded Boundary Method for Hyperbolic Conservation Laws*, Commun. Comput. **2**, pp. 1199-1219, (2007)

See also:

1. An overview of the Embedded Boundary Method and related papers can be found in the *Serpentine Wave Propagation* web-page
   `https://computation.llnl.gov/casc/serpentine/overview.html`

2. Trebotich, D. et al. *Performance of Embedded Boundary Methods for CFD with Complex Geometry*, Journal of Physics: Conference Series 125 (2008), available on-line at:
   `http://www.iop.org/EJ/article/1742-6596/125/1/012083/jpconf8_125_012083.pdf`

3. One of my previous reviews on the Cartesian cut-cell approach:
   `http://www.surengineering.com/AMS/MR1500047.pdf`