

**Title: Some reduced finite difference schemes based on a proper orthogonal decomposition technique for parabolic equations**

**Author: Sun, P., Luo, Z. and Zhou, Y.**

Published in: *Applied Numerical Mathematics*, **60**, 154–164, (2010)

Review by: Mario Forcinito

The authors present a reduced finite difference scheme based on the Crank-Nicholson scheme for parabolic equations. The reduced scheme is derived using the Proper Orthogonal Decomposition (POD) technique.

POD method has been used in several disciplines (fluid dynamics, signal processing, image analysis, structural dynamics, etc.) to extract the mode shapes or basis functions from experimental data. The basis functions are then used to create an approximation with a lower dimensionality than the original data set using Galerkin-type projections[1].

Applying the POD method to the standard Crank-Nicholson scheme, which is unconditionally stable for the equation type, a new finite difference scheme with a reduced dimensionality is obtained. The reduced scheme can be used to advance the solution with a significant reduction in the number of equations needed.

The authors applied this reduced scheme to a 1D equation and to a 2D, using an ADI scheme. Results show that the error with respect to exact equations, obtained with a lot less computational effort, is of the same order as the original Crank-Nicholson or ADI schemes.

References:

1. Park and Zak, Model Reconstruction Using POD Method for Gray-box Fault Detection, IEEE Aerospace Conference, Big Sky, MT, 2004, available on-line at:  
<http://trs-new.jpl.nasa.gov/dspace/bitstream/2014/10672/1/02-2770.pdf>