Title: **High order conservative differencing for viscous terms and the application of vortex-induced vibration flows**

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Applications that require accurate prediction of flows, such as direct numerical simulation of turbulence, benefits from high order (higher than 2nd order) finite difference schemes. There is a renewed interest for the application of high order approximations to the accurate modeling of flows in engineering problems. Accurate predictions in complex flows can be achieved only if the computational scheme has a very low numerical dissipation. As numerical dissipation is directly related to the size of the 'neglected' terms in the Taylor's series expansion, a higher order approximation implies a smaller numerical dissipation. The problem with higher order finite differences schemes is the size of the stencil needed to achieve the required order in the representation of solution.

Accurate representation when solving the Navier-Stokes equations needs high order evaluation of both, the inviscid and viscous fluxes. A great deal of the research effort has been concentrated on the development of highly accurate approximations for the inviscid terms, in particular geared to the obtention of schemes that are non-oscillatory (ENO, WENO, etc). The authors present in this paper a conservative 4th-order finite differencing scheme for the viscous terms which is applied together with a 5th-order WENO scheme to solve the compressible Navier-Stokes equations.

The authors goal is a conservative finite differencing scheme for the viscous term that is required to achieve 4th-order accuracy with a stencil that 'fits' inside the stencil for the 5th-order WENO. The authors found that these requirements are met if the solution point is located half a grid interval in each direction from the nodal points. Such characteristic makes this scheme suitable for its implementation along the lines of lower order algorithms in which the quantities are calculated at the centroid of the cell. This characteristic also simplifies the application of boundary conditions conditions over walls. The scheme was first presented in [1] and its 4th-order character is demonstrated in this paper.

The scheme is presented for its use with generalized coordinates. The paper also show the authors' solution for the problem of an elastically supported cylinder in a two dimensional flow obtained using the proposed scheme together with a 5th-order accuracy WENO scheme for the inviscid flows. In this case the solution of the flow over the cylinder and the dynamics of the cylinder as a mass-spring system acted by the (unsteady) lift and drag forces are solved simultaneously using 2nd-order backward finite differencing. The results show good agreement with experimental and numerical results by others.

References:

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   `http://www.eng.miami.edu/acfdlab/publications/AIAA-2007-4431_i.pdf`

See also:

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   `http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.32.4898&rep=rep1&type=pdf`

2. Vasilyev, O. *On the construction of high order finite difference schemes on non-uniform meshes with good conservation properties*, Center for Turbulence Research Annual Research Briefs (1998), available on-line at:
   `http://ctr.stanford.edu/ResBriefs98/vasilyev.pdf`