Title: **A Combined Finite Volume-Finite Element Scheme for the Discretization of Strongly Nonlinear Convection-Diffusion-Reaction Problems on Nonmatching Grids**

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The authors present a numerical scheme for non linear degenerate parabolic convection-diffusion reaction equations in two and three space dimensions. This type of equations appear in the context of contaminant transport in porous media with equilibrium adsorption reaction. A typical model equation for this type of problems can be written as:

$$\frac{\partial \beta(c)}{\partial t} - \nabla \cdot (\mathbf{S} \nabla c) + \nabla \cdot (c\mathbf{v}) + F(c) = q \tag{1}$$

where c is the unknown concentration, $\beta(.)$ is the function that represents time evolution and equilibrium adsorption reaction, $\mathbf{S}$ is the diffusion-dispersion tensor, $\mathbf{v}$ the velocity field and $F(.)$ a reaction function.

The approach taken by the authors to discretize Eq. 1 was to use a cell centered finite volume for the time evolution, convection reaction and source term, while a conforming piecewise linear finite element applied to a simplicial dual mesh of the original grid was used to discretize the diffusion term.

In this way, the resulting scheme is fully consistent and the discrete solution is naturally continuous across interfaces with no matching grids without the need of interpolation.

The main difference between the standard mixed finite element-finite volume approaches and this work is that in this case the authors started with a non-matching finite volume grid and create a dual triangular finite element grid. This approach Leads to a simple and consistent scheme one for which there is no need of interpolation at the grid interfaces.

The authors describe the implications of this type of grid handling in detail. Convergency existence and uniqueness of the discrete solutions are proven using A-priori estimates. The results of a numerical experiment are presented to to illustrate the behavior of this scheme over non-matching grids.