Title: **Equivalence of Semi-Lagrangian and Lagrange-Galerkin Schemes under Constant Advection Speed**

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Semi-Lagrangian methods have been employed since the early 1960's in the framework of Numerical Weather Prediction and they are well established in the field of Plasma Physics. They are based on the characteristics solution lines of hyperbolic PDEs and one of their advantages is that they can be implemented as high-order, large time-step schemes not restricted by the CFL condition.

Addressing the same situations, Lagrange-Galerkin methods have been growing in popularity in applications such as CFD in the classical finite element setting.

The author has compared both approaches in the context of the constant coefficient advection evolution equation:

$$\begin{aligned} v_t(x,t) + a \cdot \nabla v(x,t) = 0, \quad & in\ \mathbb{R}^N \times \mathbb{R} \\ v(x,0) = v_0(x), \qquad & in\ \mathbb{R}^N \end{aligned} \tag{1}$$

finding that under the conditions of constant spacing mesh and constant advection speed, equivalence between the two approaches can be demonstrated. To prove this equivalence, the authors is based on the finding that it is possible to find a suitable basis for the Lagrange-Galerkin scheme that makes it coincident with the Semi-Lagrangian scheme.

The same methodology was used to analyze interpolation schemes such as point collocation, Langrange interpolation of odd order, up to the 13^{th} order, interpolatory wavelet and splines.

See also:

1. Staniforth, A. and Côté, J., *Monthly Weather Review 119 (1991) 2206-2223*, available on-line at:
`http://www.cs.cmu.edu/~djames/pbmis/Staniforth.pdf`